Wind Farm Layout Optimisation Problem

Aida Manzano Kharman, Josh Moody, Sean Bazanye-Lutu

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Abstract

The aim of this report is to optimise the layout and number of wind turbines placed in a given area for an offshore wind farm. This is done by formulating the variables, constraints and objective function, carrying out a monotonicity analysis on the constraints and relaxing the inactive constraints to simplify the model. Then, using sequential quadratic programming, a gradient based algorithm, the optimisation problem was formulated in MATLAB. The results show that it's possible to increase power output of a typical offshore wind farm by 3% by optimising the distribution and number of wind turbines in a wind farm.

1 Introduction

There's a growing demand for renewable energy sources as awareness of the urgency of climate change increases. Using wind turbines for energy generation is very common, and when it's done it often makes economic sense to place a collection of them on the same area of land. The issue that arises from this is that placing a wind turbine downwind from another reduces the maximum power output it can achieve, due to the wake effect. [2] The wake effect refers to the way in which a wind turbine takes energy out of the wind and so creates a region behind it with reduced wind speed.[2] Power output from a turbine is proportional to the cube of the wind speed going into it, [1] so even a small drop in wind speed can lead to a significant drop in power output. Figure 1 illustrates the relationship between wind speed and the distance directly downwind from a turbine.

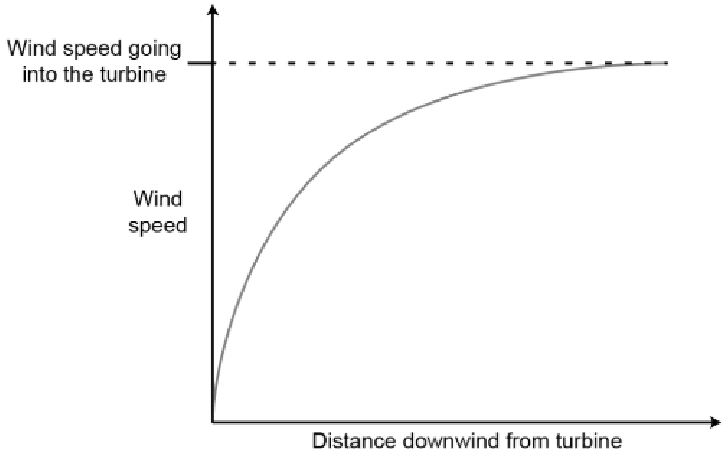


Figure 1: Wind Speed Relation to Distance

As a result of all this, ideally wind turbines would either be placed next to each other or infinitely far behind each other. This is not practical however, due to the constraints of the land available. The challenge, therefore, is to optimise the placement of turbines to maximise the total power output of a wind-farm, subject to the effect which each turbine has on the turbines downwind from it. Specifically the objective is to find, for a given plot of rectangular land, the optimal number of turbines and their optimal placement within the land which will provide the greatest total power output. It could be suggested that the optimal number of turbines would be to simply place as many as can physically fit within the space, however this is not true. There is a trade off between adding more wind turbines to increase power output, and trying to reduce the wake effect.

2 System-level problem analysis

For the system-level formulation of the problem, it was decided that the system to be optimised would be an offshore rectangular wind farm with fixed dimensions. These dimensions were chosen by using the Walney wind farm as a case study, as it is the world's largest offshore wind farm [4]. Within this enclosed area, the aim is to optimise the number of turbines and their placement inside said area, subject to the constraints that are further detailed below.

Variable Name	Meaning
n	Number of wind turbines
E_i	Energy extraction from turbine i
i	Index for each turbine, from front to back. Indexing starts at 1.
j	Index of turbine upwind of turbine i, for referencing pairs of turbines
v_i	Incoming wind speed into the turbine i
u_i	Outgoing wind speed from turbine i at some point behind it
y_{ij}	Distance between turbines i and j parallel to the prevailing wind direction
x	Distance between rows of turbines
X	Dimension of the leading edge of the wind-farm
Y	Depth of the wind farm (parallel to the prevailing wind direction)
ρ	Air density
r	Radius of rotation of the blades of the turbines
C_p	Efficiency coefficient of the wind turbines
C_T	Coefficient of thrust
α	Dimensionless scalar defining the rate of wake expansion

Table 1: Variable Names and Meanings

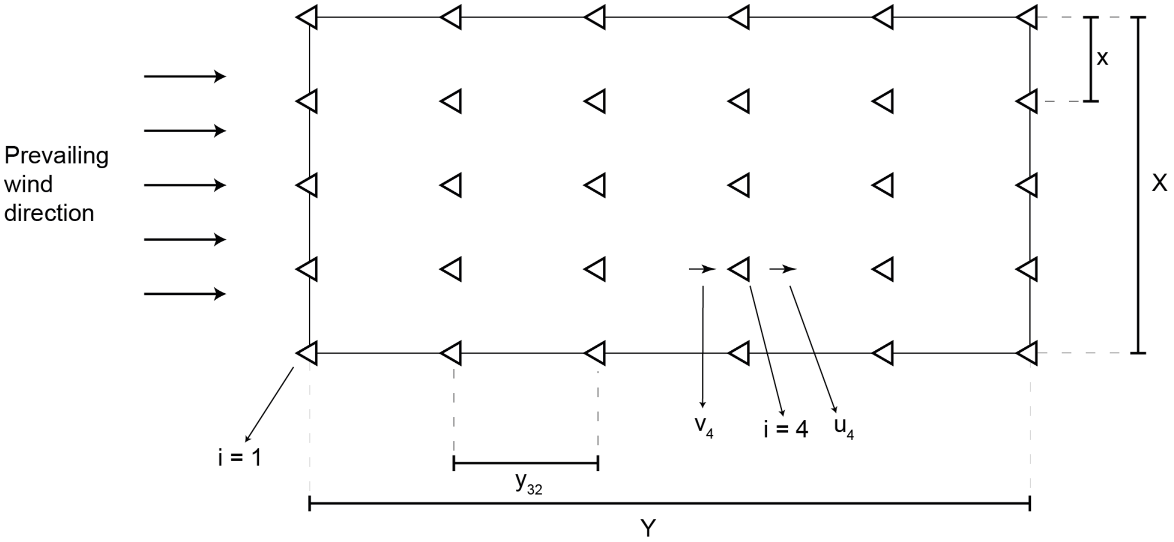


Figure 2: Visual representation of variables used in the mathematical modelling of the wind farm

$$\text{min}: \quad -P_t = \sum_{n=1}^{N} -P_i \tag{1}$$

$$\text{where}: \quad -P_i = \rho\, C_p v_i^3 \pi r^2 \tag{2}$$

$$v_i = u_j = v_j[1 - ((1 - \sqrt{1 - C_T})(\frac{r}{r + \alpha y_{ij}})^2)] \tag{3}$$

$$\text{subject to}: \quad h_1 : \rho = 1.225 \tag{4}$$

$$h_2 : v_0 = 9.26 \tag{5}$$

$$h_3 : C_\rho = 16/27 \tag{6}$$

$$h_4 : r = 75 \tag{7}$$

$$h_5 : \alpha = 0.075 \tag{8}$$

$$h_6 : x = 406 \tag{9}$$

$$g_1 : -x_i + 2r \leq 0 \tag{10}$$

$$g_2 : 1 - n \leq 0 \tag{11}$$

$$g_3 : 6r - y_i \leq 0 \tag{12}$$

$$g_4 : \sum_{i=1}^{n-1} y_i - 8.5 \leq 0 \tag{13}$$

Equation 2 describes the power produced by the i^{th} turbine, and 1 describes the relationship between the total power output of the wind farm, and the individual power output of each turbine. The equalities 4, 5 and 7 are set to the values that match those of the Walney wind farm, and are in the units of KPa, m/s and metres respectively. In particular, equation 7 refers to the rotor radius of the turbine, and it was set to 75 metres, as this is the radius of the Vestas 164 turbine, that is the most common turbine being used at the farm. Table 1 explains equation 8. Equation 10 states that the separation between turbines in the direction perpendicular to the wind flow must be 406 metres, given that this was calculated to be the minimum distance that would not create a wake effect shadow at each row of the turbines. This is illustrated in figure 3 where it can be seen that the overlap between wakes never occurs within the radius of the incoming wind to a turbine.

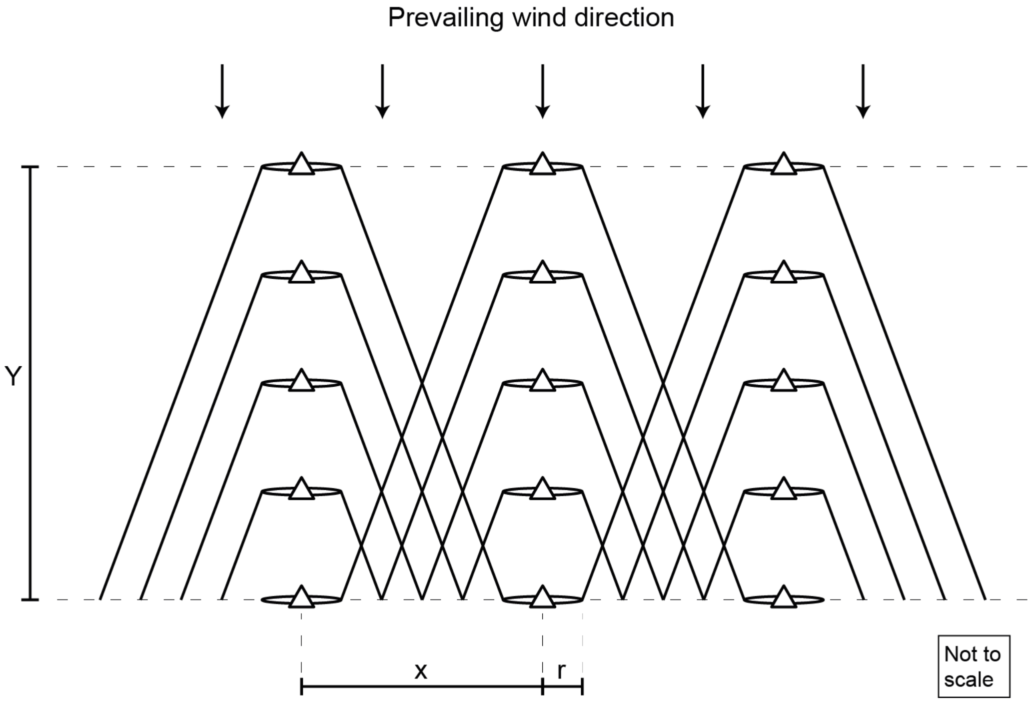


Figure 3: Using a separation of 406m between turbines, it ensures that the wake effect overlap never occurs at the point where wind flows into a turbine

Equation 9 describes that the distance between the i^{th} and the $(i^{th} - 1)$ turbines in the direction perpendicular to the wind stream must be greater than or equal to twice the rotor radius. Equation 12 specifies that the distance between the i^{th} and the $i^{th} - 1$ turbines in the direction parallel to the wind stream must be greater than or equal to six time the rotor diameter. Both of these constraints are considered approximate industry standards for efficiency and safety [3]. Equation 13 states that the sum of all the distances between turbines in the direction that is parallel to the wind stream must be smaller or equal to 8.5km, which is the estimated total width of the Walney wind farm area.

3 Modelling Approach

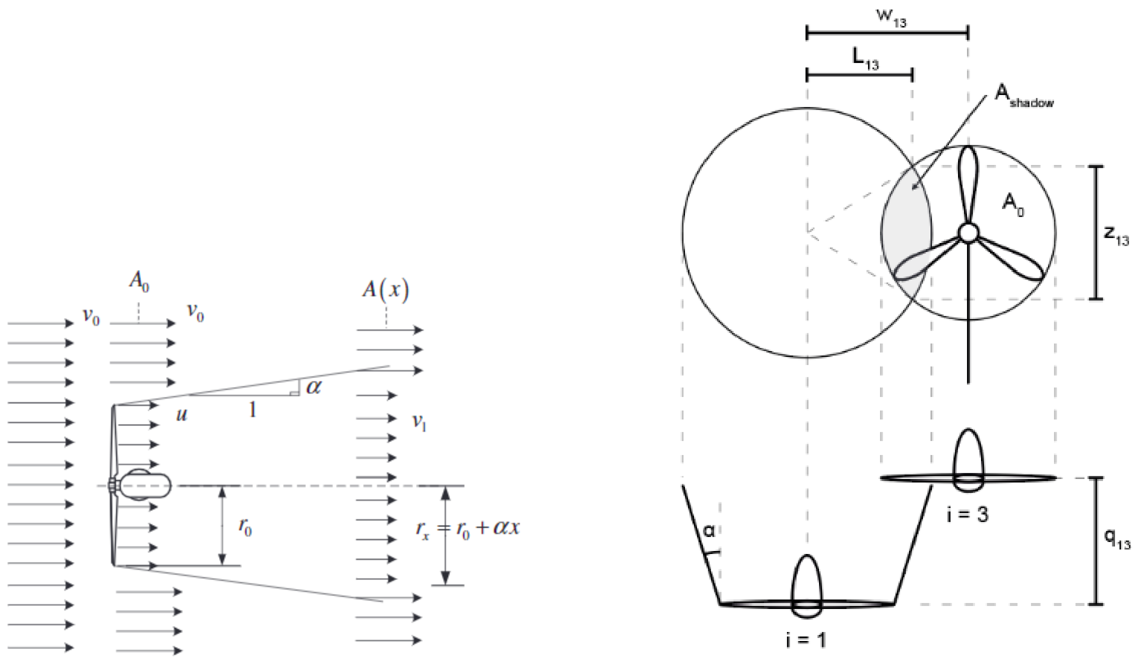
For this optimisation problem, in order to predict the optimum number of wind turbines and their layout within a farm of a given area, it was necessary to calculate the power output of each turbine, which is a function of the incoming wind velocity, as seen in equation 2. In order to calculate this incoming wind velocity, the outgoing wind velocity from the turbine in front of it must be calculated. To do this, a method to model the wake effect behind each turbine is required.

Numerous works in this field have been conducted to describe the wake effect, but it was decided to use the Jensen model for this project, however ultimately this work was informed by that of Francisco González-Longatt, who further developed the Jensen model by deriving equations which would describe the effect that the wake had on turbines as a result of those placed upwind of them. [2] The reason why this model was chosen was because adding a singular wake coefficient does not account for the wake expansion as the wind travels further away from the turbine. This expansion can be modelled in a linear manner as follows:

$$r = r_0 + \alpha x \tag{14}$$

$$\alpha = \frac{1}{2\ln(\frac{z}{z_0})} \tag{15}$$

where α determines how quickly the wake expands with distance, [2], [5] and variable z can be visualised in Figure 4b



(a) Visual representation of the linear wake model used.

(b) Visual representation of shadow effect of the interfering wakes.

Figure 4: Models being used for the optimisation problem.

This model is incorporated in equation 3 in the form of a ratio between the initial radius of the turbine, r_0, which is also the radius of the wake cone, and the final radius of the wake effect cone, which is equation

14. This was the improvement from the first model used at the interim review, where the wake effect was modelled to be a single coefficient.

4 Problem Space

To simplify the model and understand better the problem space, a monotonicity analysis was carried out on the inequality constraints of the problem shown below:

$g_1 : -x_i + 2r \leq 0$
$g_2 : 1 - n \leq 0$
$g_3 : 6r - y_i \leq 0$
$g_4 : \sum_{i=1}^{n-1} y_i - 8.5 \leq 0$

	y_i	n
f	-	-
g_1		
g_2		-
g_3	+	
g_4	+	+

The objective function is equation 1, and as shown by equation 2, the only variables in that equation are the number of turbines n and the velocity of the incoming wind v_i, and the rest of the terms are constants which are defined by the equalities in the problem formulation. Equation 3 shows there is a relationship between y_i, the distance between turbines, and the incoming wind speed, and Figure 1 proves that as the wind speed increases so does y_i, and as such, when formulated in the negative-null form, y_i monotonically decreases as the objective function is minimised. n also monotonically decreases as the objective function is minimised, given that the more wind turbines added, the higher the total power output of the field will be, and so it is monotonically decreasing as the objective function in the negative-form is minimised.

Starting with the n variable, it can be seen that the only active constraint is g_4. This is because g_4 states that the sum of all the individual distances between turbines i and $i-1$ from turbine 1 to turbine N, in the direction that is parallel to the wind flow, must be smaller or equal to 8.5km. As you increase the number of turbines n, there will be more y_i terms to add to the summation, and therefore the total sum would increase. This therefore explains why g_4 monotonically increases as n is increased. However, for this condition to remain true, n must always be a positive number, because theoretically, having a negative number of n would decrease the total sum of all y_i terms. n must also always be a positive number because it is physically impossible to have a negative number of turbines, so therefore, another constraint must be added for g_4 to be monotonically increasing in the n variable, which is: $g_5 : n \geq 0$. However, given the nature of the optimisation problem, adding zero turbines to the wind farm is not a valid solution, meaning that constraint $g2$ is more constraining, as $1 > 0$. Therefore, in order for the problem to be well constrained in the n variable, constraint g_4 must be active, and to ensure that g_4 is monotonically increasing in n, g_2 must also be active.

In terms of the y_i variable, it can also be seen that g_4 is active as it monotonically increases as y_i is increased. However, this is only the case as long as y_i remains positive. As such, another constraint g_6 must be introduced stating that $g_6 : y_i \geq 0$. However, because it has been established that zero number of turbines cannot be part of the solution, then it is not possible that the distance between them y_i is zero as well. Furthermore, industry safety standards dictate that turbines should be places at least six times their radius apart in the y_i direction, which is what constraint g_3 states. Because $6r > 0$, g_3 is more constraining than g_6, and as such, it must be active too. To summarise, for the y_i variable to be fully constrained in this optimisation problem, both g_4 and g_3 must be active. This allows for constraint g_1 to be eliminated and thus simplifies the problem.

5 Optimisation Process

A gradient-based optimisation method was used for this project, as it was expected that it would yield better results both regarding efficiency and accuracy, due to it being a convex problem. Sequential quadratic programming was implemented in MATLAB, using the sqp algorithm with the fmincon() function.

There was a unique challenge faced optimising this problem, in that the formulation of our objective function depends on the variable n, the number of turbines to be placed, as shown in equation 1 and 2. In order to overcome this, the optimisation problem was run multiple times for a range of different values of n, and then the one which achieved the greatest total power output was selected for the final result, along with the corresponding values of y_i for i = 1 to n.

6 Discussion

One of the challenges faced throughout this optimisation was efficiently modelling the problem. As turbines in a farm experience wind from multiple different sources upstream, it resulted too complex and labour intensive to try and correctly model this relationship. It was identified that on the scale of an actual wind farm, the region affected by the wake shadowing effect of the turbines upstream was quite small, in fact it was smaller than the typical wind turbine distribution currently used. Therefore consideration did not need to be paid to this as long as turbines where placed outside this region.

As a result of this, the challenge was to optimise the distribution and number of turbines in each row of the wind farm, where each row was to be laid parallel to the prevailing wind direction. These rows could then be distributed far enough apart from each other so that the wake effect from one row of turbines would not affect the rows next to it. The final optimised wind farm is shown in figure 6.

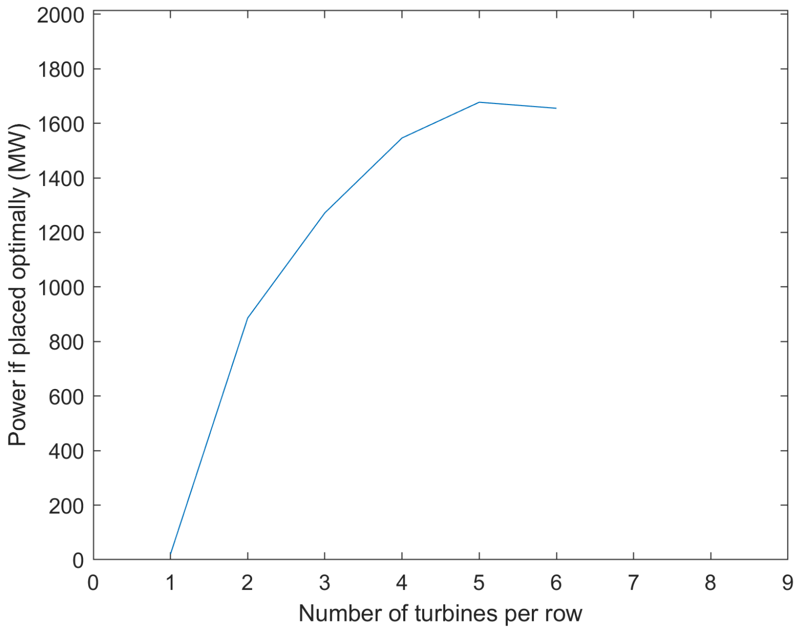


Figure 5: Power production of the field

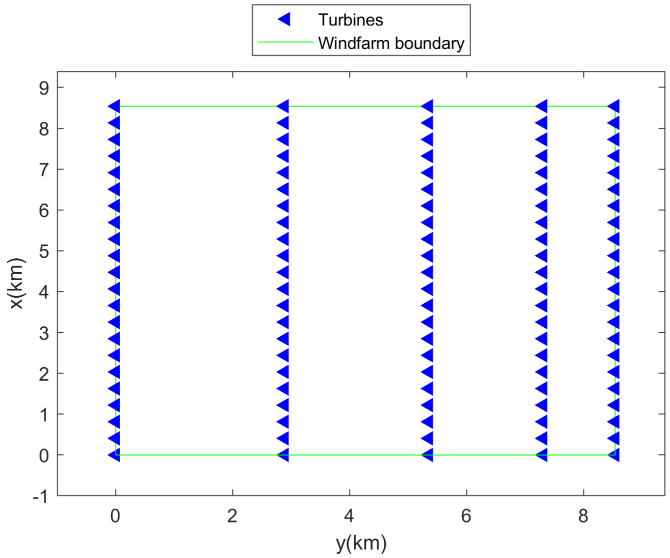


Figure 6: Optimised wind farm turbine layout

It can be see from Figure 4 that more turbines doesn't necessarily yield greater power. As shown, more than 5 turbines actually reduces the total power produced by the farm. This can be explained by Figure 5. We noticed that as you travel further downstream, the spacing between turbine rows decreases. This can be attributed to the turbulent wind profile produced by the turbines upstream. As the turbine rows are placed out the wake shadow region, the only influence on the wind speed a turbine experiences is the turbines in front of it. Turbulent wind reaches its maximum velocity faster than laminar wind, and as the model's aim is to maximise power production, the location of the turbines directly coincides with areas of maximum wind speed. However, as shown in Figure 1, the rate of increase of wind speed plateaus the greater the distance. Due to this, in order to maximise the power produced, not every turbine placed is generating maximum power. This is because there is more benefit adding in an additional turbine than increasing the distance downstream once towards the end of the wind speed profile. Adding too many turbines, however, results in more turbines being forced outside the maximum wind speed region and thus, reduces the overall power produced.

Parametric studies were carried out for both the turbine radius and the ambient wind speed to assess the impact which these had on the problem and to see if there was opportunity to further optimise the solution. It was found that the turbine radius was proportional to the total power output of the wind farm, as shown in figure 7. This fact would be useful for developers choosing which turbine size to use on a wind farm, to see how to get the best value for money.

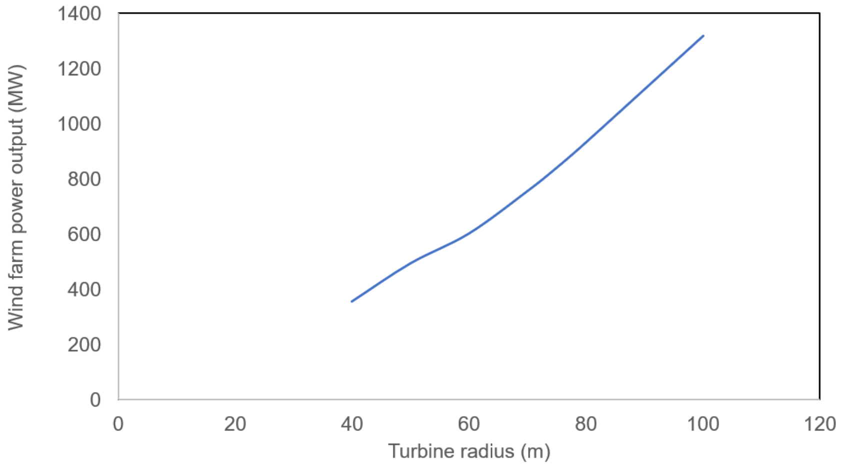


Figure 7: Relationship between turbine radius and wind farm power output

The results of a parametric study conducted on ambient wind speed are shown in figures 8 and 9. There are a number of insights gained from this data. Firstly, wind farm power output increases exponentially with ambient wind speed. Secondly, the number of wind turbines which produce the optimal power output changes greatly with ambient wind speed, in particular when it is very low. This data can be put to use to get the most value out of a wind farm. For example, if there's an ambient wind speed of around 9.5m/s, it is around the point at which 6 turbines would be able to generate more power than 5. Nonetheless, placing these 6 would not be taking advantage of their full potential, whereas placing 5 would. Placing 6 might be more advantageous, however, only if there were a positive skew on the wind speed probability distribution. This is because on the occasional times when the wind speed gets significantly higher, the 6 turbines will be able to have a much greater output than the 5 could. Under the same reasoning, it would not be as advantageous to to place 6 turbines if there were a negative skew.

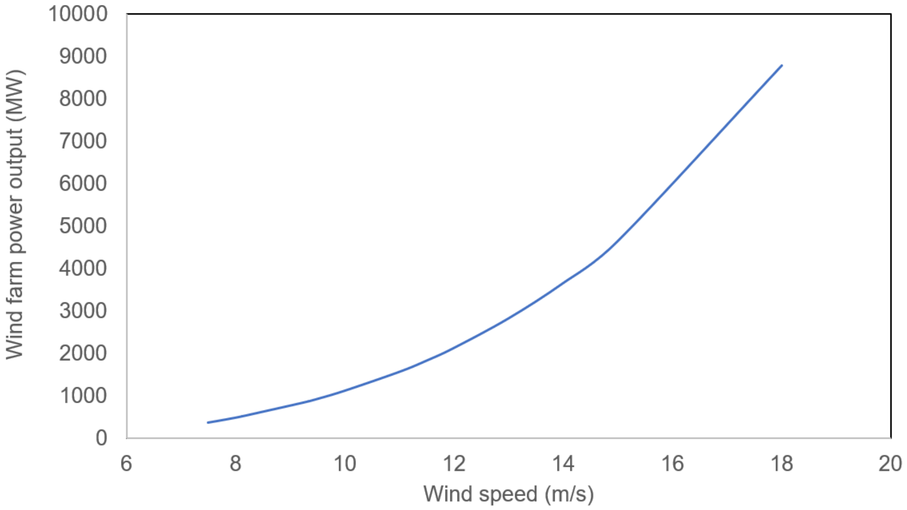


Figure 8: Relationship between ambient wind speed and wind farm power output

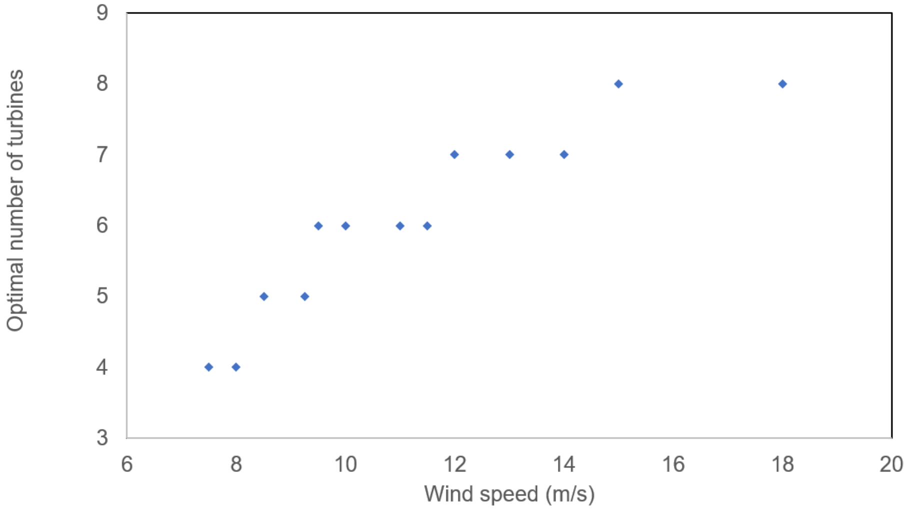


Figure 9: Relationship between ambient wind speed and optimal number of wind turbines

7 Conclusion

Through modelling of the wake effect it was possible to describe the wind speed behind a turbine. Using this, an objective function and a series of constraints were developed to optimise the distribution and number of turbines in a row on a wind farm. The entire wind farm could therefore be optimised to produce the maximum possible power with the space available under certain conditions, by creating many rows of turbines distributed evenly apart to eliminate the wake effects between rows. Doing this for an offshore wind farm modelled on Walney wind farm, it was found that the optimal number of turbines per row was 5, and with there being 11 rows meant there were 55 turbines used in total. This yielded a total power output of 1677 megawatts for the entire farm. If the turbines were distributed evenly it's expected they would produce 1626 megawatts. This means the optimised solution is increasing power output by 3%.

One clear opportunity to improve this model would be the inclusion of cross-winds. If wind coming at angles to the turbines could be effectively modelled, turbines could be placed in the current dead zones, allowing the overall power generated by the wind farm to potentially increase. Another improvement which could be made would be to make the turbine radius a variable rather than a constant, allowing for different types of wind turbine models to be considered, as it is often the case that wind farms use more than one wind turbine model. This would thereby widen the problem space to allow even more optimal solutions to be found.

8 Author contributions

Throughout the project all members conducted research into the field and worked jointly to describe and develop the methods and models used in this report. The report was co-written by all members.

Josh Moody Contribution: Researched different existing models, worked on writing parts of the code, created graphics and wrote various sections within the report.

Sean Bazanye-Lutu Contribution: Researched and prototyped 2 dimensional modelling methods, worked on writing parts of the code, wrote the discussion section of the report

Aida Manzano Kharman Contribution: Wrote various sections of the report, in particular the abstract, monotonicity analysis, modelling approach and problem space sections. Worked on the layout, referencing and final proofreading.

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