Introduction:

The design problem assigned for this task was the modelling of a tandem bicycle frame that is lightweight, unisex, with natural frequencies above 30Hz to avoid discomfort and with an effective life of at least 10 years.

Through the use of FEA the last two criterion were tested in order to ascertain that the designs were successful in passing all the required standards.

Methods:

Modelling Assumptions:

The modelling assumptions used were that the structural elements used for the tandem bicycle frame are beam elements, and as such, according to Timoshenko's beam element theory, their cross section remain planar and rotate with respect to the neutral axis only. Also, according to this theory, it is assumed that the curvature of the beam is very slight and that the length is the largest dimension out of the three.

Furthermore, it is assumed that the system is linear, meaning that the way in which elements' coordinates relative to each other are interpolated uses the following relationship:

Where x, y and z are:

$x = \sum N_i x_i i \quad y = \sum N_i y_i i \quad z = \sum N_i z_i i$

And the 4-node tetrahedrons are:

$N_1 = 1 - r - s - t \quad N_2 = r \quad N_3 = s \quad N_4 = t$

The relationship between r, s, and t is linear and is generally a suitable way of approximating shape functions that do not have many curves in them, like the tandem bicycle frame. (Ghajari, 2019)

Boundary Conditions:

The boundary conditions used were assuming that the inner surface of the fork shell is fixed and that the rear wheel bearing is hinged. The physical implication of fixing the fork shell is that for a beam, it sets the three translational and the rotational degrees of freedom to zero, as opposed to an immovable fixture that only sets the translations to zero. (Dassault Systemes, 2012)

The hinge fixture applies a radial restraint=0 and an axial restraint=0 to the nodes of the selected entities, meaning that the cylindrical face can only rotate about its own axis. This is a suitable way of modelling of the wheel shaft turning axially in the shell.

Loads:

The loads applied were 981 N normal to the faces depicted below to model two humans weighing 100kg sitting on them.

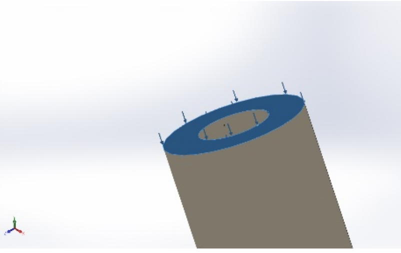


Figure 1: Face where human weight is applied

For the force applied at the pedals, the loads applied were modelled as remote loads acting on the inner face of the crank shells. Using the centre of the face of the crank shells as the new reference frame, the load was set to act at 200mm from the centre in the x direction, and at 100mm in the z direction. Because it was an oscillating force between 0 and 1kN, the force was set to 1kN downwards in the y direction as shown below.

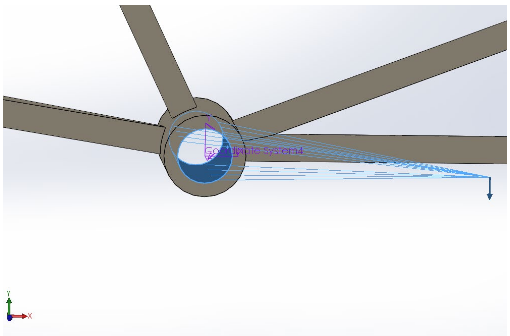


Figure 2: Crank shell face where remote force is applied

Simulations:

For each model iteration, there were four simulations carried out to asses how well it met the criteria of the design problem. These were two static simulations, a frequency and a fatigue simulation.

Static:

For the static simulations, it is assumed that the system's variations are so small over time that they are negligible, and as such the equilibrium equation of the structure can be described as $[\mathbf{K}]\{\mathbf{D}\} = \{\mathbf{R}\}$, where K is the structure stiffness matrix and $\{\mathbf{D}\}$ and $\{\mathbf{R}\}$ are the nodal displacement vector and nodal forces vector. (Ghajari, 2019)

The stress tests were performed using the Von Mises criterion, because it is the test used for ductile materials, where if $\delta_v > S_y$ then the material fails, $\delta_v = \sqrt(\frac{(\delta_1-\delta_2)^2+(\delta_2-\delta_3)^2+(\delta_3-\delta_1)^2}{2})$ and S_y is the stress at yield. Since the materials being analysed are aluminium alloy and magnesium alloy, both ductile materials, it is therefore a suitable test to utilise.

The fixtures used were the ones described above, and so were the loads modelling the humans' weight. The difference between each static study was the side of the frame on which the remote loads were acting. For the first simulation, the loads were set to act on each side of the frame, whereas for the second simulation they were both acting on the same side.

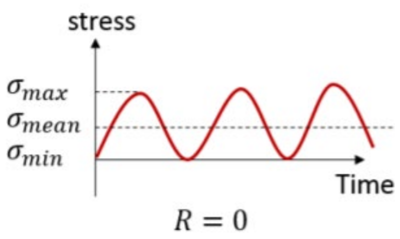
The reason for this was to model how the frame behaves under varying load conditions, and to test if the design has any flaws or asymmetries that would result in large displacements or internal stress concentrations if the loads applied are not symmetrical. It was also a necessary tool used as a sanity check, given that if the results of each simulation are vastly different, it would be a clear sign of a problem with the way in which the simulation was set up and modelled.

Fatigue:

Given that most materials in real situations fail at stress levels smaller than the yield or ultimate strength of the material, it is necessary to carry out fatigue studies to test the amplitude of cyclic stress that the model can withstand without causing it to fail.

This is because under daily use, the loading is not typically constant as depicted in many stress strain curves, and therefore testing the model with cyclic loading is more reflective of what its performance will be like.

Having completed the two static simulations, these were then used in the fatigue simulation as separate events with a loading cycle of 1e+06 as specified by the brief. The loading selected was zero based, because the oscillation of the force from the pedals goes from 0 to 1kN, and therefore the loading of the bicycle frame will be more accurately represented by the graph below.



In this case $\delta_{max} = 1kN$ and $\delta_{min} = 0$

Figure 3: Type of cyclic loading used

(Ghajari, 2019)

Another parameter that was significant was the alternating stress used. Because usually the S-N curves available have been calculated using a R= -1 loading condition (meaning the load oscillates from a mid-point value of zero and the maximum amplitudes are positive and negative), it is necessary to correct the alternating stress value. This was done using the Gerber method, given that this is the suitable method to use for a ductile material.

$$S_{ca} = \frac{S}{1 - (\frac{\delta_{mean}}{S_u})^2}$$

Where S_{ca} is the corrected alternating stress and S_u is the ultimate stress.

Frequency:

Regarding the frequency simulations, these were done to model the system as a dynamic one as opposed to a static one. As opposed to using $[\mathbf{K}]\{\mathbf{D}\} = \{\mathbf{R}\}$ as the FE equation, $[\mathbf{M}]\{\ddot{D}\} + [\mathbf{C}]\{\dot{D}\} + [\mathbf{K}]\{\mathrm{D}\} = \{\mathbf{R}\}$ is more reflective of the behaviour of the system under dynamic loading, where M is the mass matrix, C is the damping matrix and K is the stiffness matrix, and D are the vectors of nodal displacements, with $\ddot{D}$ and $\dot{D}$ being the first and second time derivatives.

From Newton's second law of motion, we can derive the following equation:

$m\ddot{x} + c\dot{x} + kx = f(t)$

The force f(t) in this case is oscillatory, and if the system is undergoing undamped free oscillations, then the equation becomes:

$m\ddot{x} + kx = f(t)$ where $x = A\ sin\omega\ t$ and $\omega = \sqrt{\frac{k}{m}}$

As such, it is possible to find the natural frequency, and through the use of these derivations, making changes in the design to increase the natural frequency is easier. The aim is to have a natural frequency higher than 30Hz, and therefore, a higher k and a lower m will provide the most desired outcome.

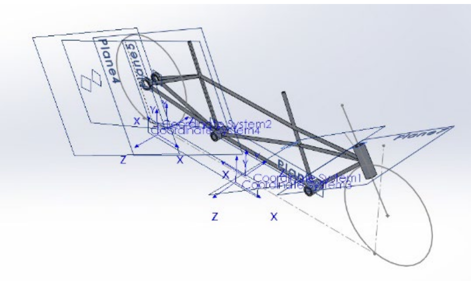
(College Physics, 2017)

Results:

Aluminium alloy:

First Iteration:

20mm outer diameter and 5mm wall thickness



The mesh size displayed was the default one used for all models before applying mesh refinement

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	5 mm
Tolerance	0.25 mm
Mesh Quality Plot	High

Static Study 1:

Name	Type	Min	Max
Stress	VON: von Mises Stress	3.016e+00 N/m^2 Node: 25073	1.892e+08 N/m^2 Node: 193901
Displacement	URES: Resultant Displacement	0.000e+00 mm Node: 1229	1.077e+01 mm Node: 24235
Strain	ESTRN: Equivalent Strain	1.145e-10 Element: 107020	1.939e-03 Element: 57550

Static Study 2:

Name	Type	Min	Max
Stress	VON: von Mises Stress	6.487e+00 N/m^2 Node: 207424	1.822e+08 N/m^2 Node: 197985
Displacement	URES: Resultant Displacement	0.000e+00 mm Node: 1229	1.313e+01 mm Node: 24235
Strain	ESTRN: Equivalent Strain	3.807e-10 Element: 23859	1.992e-03 Element: 26699

Frequency Study:

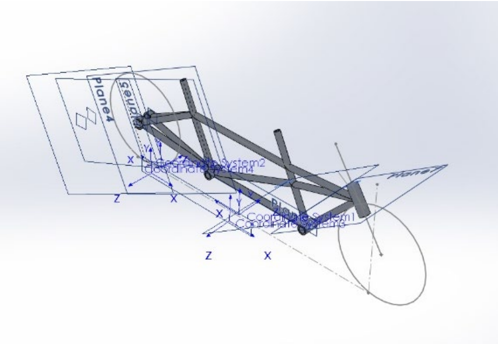
Mode Number	Frequency (Hertz)	X direction	Y direction	Z direction
1	**38.518**	**3.1856e-09**	**6.2396e-08**	**0.37961**
2	**47.157**	**2.2754e-09**	**7.1449e-08**	**0.084605**
3	**91.358**	**4.6441e-11**	**2.2393e-08**	**0.039363**
4	**120.6**	**1.782e-08**	**2.5228e-06**	**0.018785**
5	**124.65**	**0.00012592**	**0.030041**	**8.0989e-07**
		Sum X = 0.00012594	**Sum Y = 0.030044**	**Sum Z = 0.52236**

Fatigue Study:

Name	Type	Min	Max
Results	Life plot	3.026e+06 cycle Node: 197985	4.000e+07 cycle Node: 1

Second Iteration:

40mm outer diameter and 10mm wall thickness



Static Study 1:

Name	Type	Min	Max
Stress	VON: von Mises Stress	3.449e+01 N/m^2 Node: 2817	3.785e+07 N/m^2 Node: 369068
Displacement	URES: Resultant Displacement	0.000e+00 mm Node: 1350	7.056e-01 mm Node: 49935
Strain	ESTRN: Equivalent Strain	2.671e-10 Element: 85314	4.252e-04 Element: 21820

Static Study 2:

Name	Type	Min	Max
Stress	VON: von Mises Stress	2.036e+01 N/m^2 Node: 221536	3.421e+07 N/m^2 Node: 553781
Displacement	URES: Resultant Displacement	0.000e+00 mm Node: 1350	9.771e-01 mm Node: 49935
Strain1	ESTRN: Equivalent Strain	2.997e-10 Element: 28472	3.808e-04 Element: 26026

Frequency Study:

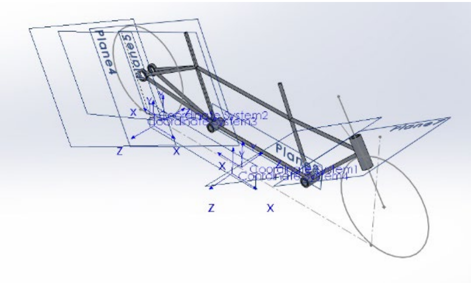
Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	74.495	5.6843e-12	4.1552e-10	0.55431
2	102.63	3.7072e-11	4.1195e-10	0.022244
3	171.04	6.5147e-10	1.6677e-12	0.01077
4	193.37	6.7699e-10	1.946e-09	0.046682
5	226.38	0.012665	0.40591	4.2477e-09
		Sum X = 0.012665	Sum Y = 0.40591	Sum Z = 0.63401

Fatigue Study:

Name	Type	Min	Max
Results1	Life plot	4.000e+07 cycle Node: 1	4.000e+07 cycle Node: 1

Third Iteration:

No supporting beams in the centre of the frame.



Static Study 1:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	1.183e+01 N/m^2 Node: 1533	1.671e+08 N/m^2 Node: 176480
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1	1.263e+01 mm Node: 22488
Strain1	ESTRN: Equivalent Strain	3.771e-10 Element: 56492	1.825e-03 Element: 18081

Static Study 2:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	1.165e+01 N/m^2 Node: 196699	2.011e+08 N/m^2 Node: 180638
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1	1.427e+01 mm Node: 22506
Strain1	ESTRN: Equivalent Strain	4.252e-10 Element: 17141	2.260e-03 Element: 27155

Frequency Study:

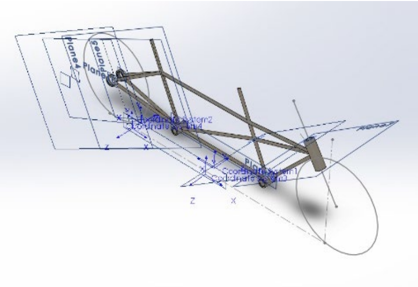
Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	36.052	3.5966e-09	3.9761e-08	0.35624
2	38.488	1.8998e-08	4.2585e-08	0.025506
3	98.393	5.157e-10	3.8017e-08	0.080828
4	110.75	3.7421e-05	0.029204	1.9673e-08
5	116.33	7.7179e-08	5.6575e-06	0.0019548
		Sum X = 3.7521e-05	Sum Y = 0.02921	Sum Z = 0.46453

Fatigue Study:

Name	Type	Min	Max
Results1	Life plot	2.154e+06 cycle Node: 178310	4.000e+07 cycle Node: 1

Magnesium Alloy:

First Iteration:



Static Study 1:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	4.229e+00 N/m^2 Node: 86118	1.880e+08 N/m^2 Node: 193901
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1229	1.737e+01 mm Node: 24235
Strain1	ESTRN: Equivalent Strain	1.495e-10 Element: 57755	3.148e-03 Element: 57550

Static Study 2:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	9.673e+00 N/m^2 Node: 207424	1.823e+08 N/m^2 Node: 197985
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1229	2.114e+01 mm Node: 24235
Strain1	ESTRN: Equivalent Strain	7.301e-10 Element: 23859	3.239e-03 Element: 26699

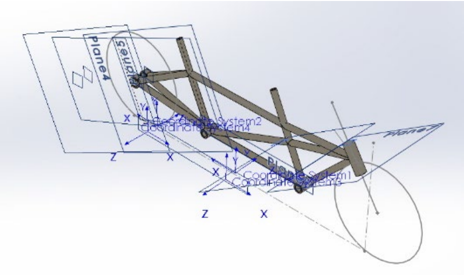
Frequency Study:

Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	40.666	7.6137e-09	1.3464e-07	0.39283
2	45.988	5.5881e-09	1.7507e-07	0.072357
3	94.226	6.7854e-11	6.996e-08	0.040395
4	120.52	2.2792e-08	1.082e-05	0.014571
5	123.96	0.00012828	0.030251	2.6146e-06
		Sum X = 0.00012832	Sum Y = 0.030262	Sum Z = 0.52016

Fatigue Study:

Name	Type	Min	Max
Results1	Life plot	2.957e+06 cycle Node: 197985	4.000e+07 cycle Node: 1

Second Iteration:



Static Study 1:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	3.371e+01 N/m^2 Node: 562876	3.763e+07 N/m^2 Node: 369068
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1350	1.137e+00 mm Node: 49935
Strain1	ESTRN: Equivalent Strain	4.825e-10 Element: 85314	6.896e-04 Element: 21820

Static Study 2:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	2.140e+01 N/m^2 Node: 221536	3.437e+07 N/m^2 Node: 553781
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1350	1.572e+00 mm Node: 49935
Strain1	ESTRN: Equivalent Strain	8.517e-10 Element: 85314	6.215e-04 Element: 26026

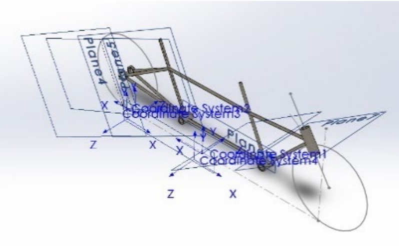
Frequency Study:

Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	75.912	1.1869e-12	1.2436e-09	0.55506
2	103.8	4.8285e-11	1.9295e-09	0.021349
3	174.14	8.7561e-10	1.2497e-10	0.010474
4	196.07	1.029e-09	1.8143e-09	0.047015
5	230.17	0.012542	0.40452	4.3857e-09
		Sum X = 0.012542	Sum Y = 0.40452	Sum Z = 0.6339

Fatigue Study:

Name	Type	Min	Max
Results1	Life plot	4.000e+07 cycle Node: 1	4.000e+07 cycle Node: 1

Third Iteration:



Static Study 1:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	1.116e+01 N/m^2 Node: 1533	1.667e+08 N/m^2 Node: 176480
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1	2.036e+01 mm Node: 22488
Strain1	ESTRN: Equivalent Strain	5.810e-10 Element: 56492	2.955e-03 Element: 18081

Static Study 2:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	1.258e+01 N/m^2 Node: 196699	2.011e+08 N/m^2 Node: 180638
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1	2.296e+01 mm Node: 22506
Strain1	ESTRN: Equivalent Strain	7.767e-10 Element: 17141	3.672e-03 Element: 27155

Frequency Study:

Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	32.972	2.1407e-08	2.6305e-08	0.1599
2	38.694	2.7653e-08	1.4171e-07	0.22214
3	101.16	1.5117e-12	1.4543e-06	0.079844
4	106.93	5.8142e-05	0.03087	4.7714e-06
5	113.45	2.1756e-07	1.5263e-05	0.00043347
		Sum X = 5.8408e-05	Sum Y = 0.030887	Sum Z = 0.46232

Fatigue Study:

Name	Type	Min	Max
Results1	Life plot	2.111e+06 cycle Node: 178310	4.000e+07 cycle Node: 1

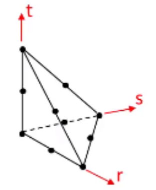
Sanity checks:

The resulting forces that the simulation returned were within a reasonable range, all in the units of MPa and none of the displacements surpassed the scale of mm.

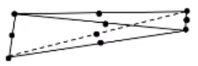
Both static studies returned results within an acceptable level of difference with no major discrepancies in any pair.

All studies were also checked for resultant forces, and the results were rational. As a rough estimate, if the sum of both humans' resultant force is said to be 2000N, and they each exert a 1000N force with their pedals on the frame, the expected resultant force should be close to 4000N. In fact, the simulation states that the reaction forces are approximately 3400N and given that the prior estimate was an over estimate, then the result seems reasonable.

Looking at the mesh quality, 99.5 % of the elements have an aspect ratio below 3, meaning that only a very small percentage of elements have been highly distorted, and the majority of the elements resemble more the one depicted on the left as opposed to the one on the right.



An ideal element with aspect ratio one (Parent element)



An element with high aspect ratio

Mesh refinement study results: (of successful model)

Displayed below are the results of mesh refinement studies of the selected successful model. Both static studies indicated that the elements with highest stress concentrations were the beams connected to the crank shells, and as such mesh refinement was applied to those faces.

The mesh size used for the rest of the body remained at 5mm and decreased to 3mm for the faces where mesh control was applied.

After four iterations of decreasing the mesh size in intervals of 0.5mm, the final results converged to the following:

Static Study 1:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	9.297e+00 N/m^2 Node: 171108	2.190e+08 N/m^2 Node: 355353
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1252	1.738e+01 mm Node: 39906
Strain1	ESTRN: Equivalent Strain	1.565e-10 Element: 49113	3.733e-03 Element: 26342

Static Study 2:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	1.117e+01 N/m^2 Node: 367437	2.273e+08 N/m^2 Node: 357836
Displacement1	URES: Resultant Displacement	0.000e+00 mm Node: 1252	2.115e+01 mm Node: 39906
Strain1	ESTRN: Equivalent Strain	8.146e-10 Element: 151742	4.004e-03 Element: 25320

Frequency Study:

Mode Number	Frequency(Hertz)	X direction	Y direction	Z direction
1	40.654	7.5898e-09	1.3673e-07	0.39269
2	45.977	5.3536e-09	1.7585e-07	0.072424
3	94.204	3.0875e-11	6.9696e-08	0.040464
4	120.5	2.3042e-08	1.0457e-05	0.014582
5	123.93	0.00012771	0.030332	2.6088e-06
		Sum X = 0.00012775	Sum Y = 0.030342	Sum Z = 0.52017

Discussion:

Iteration rationale:

Taking these considerations discussed in the method section into account, in order to improve the natural frequency value to be above 30Hz, the first iteration of the model had a wider diameter of the tubes, and a thicker inner wall. This means it has a higher moment of inertia, and because frequency of oscillation and moment of inertia are proportional, then this results in a higher natural frequency of the frame. (Myant, 2018) This was then corroborated by the experimental data.

The second iteration had less beam members in order to reduce the mass and thus increase the natural frequency. The k parameter was changed through the use of different materials, which in this case were aluminium alloy and magnesium alloy. Magnesium alloys have a lower stiffness than aluminium alloys (Science Direct, 2013) so it is expected that they will have lower natural frequencies, however, they are much lighter than aluminium alloys, so overall, they still achieve higher natural frequencies.

Most successful model:

First Iteration Magnesium Alloy

Looking closely at the results, it was found that the first iteration in magnesium alloy was the most successful tandem bicycle frame.

Both static studies showed that the maximum stresses differed by approximately 3% meaning that regardless of where the pedal force is applied the performance is consistent.

The natural frequency is 40.7 Hz which is sufficiently high to surpass 30Hz and is an improvement from the aluminium alloy equivalent model which was 38.5Hz, and the second magnesium iteration that was 33.0Hz. Although the improvement in frequency in this model is not significantly high, this model was still selected over other models with frequencies around 75Hz because of compromise with weight. This particular model is very light, approximately weighing 4.25kg, over the aluminium contender that weighed 7.01kg.

Its life cycle is also sufficiently high to meet the requirements, with even the minimum cycle length of this model being three times over 1e+06

Theoretical explanation of results:

It is shown through experimental data that increasing the moment of inertia of the model has more significant effects on the natural frequency that decreasing the mass. It is also shown that a frame of a higher mass and thicker wall diameter will perform better in fatigue tests and can withstand higher stresses.

Furthermore, the magnesium frame achieves higher frequencies in two out of three models. This is because magnesium is 33% lighter than aluminium, so as the mass decreases, the natural frequency increases. This supports the relationship between both derived in the methods section.

Limitations:

A possible source of error could be modelling error. Tandem bicycle frames are more complex than the over-simplified model utilised for the FEA analysis, and certain features might not be accurate. An example is the angle of the faces where the seats would be placed. In the model used they are not horizontal, and that could have an effect on the resultant forces and the maximum stresses, as well as the displacements.

Discretisation error is another error that could arise, however, the effects of this are minimised by increasing the number of elements and using mesh refinement, all of which were implemented in this study, and as such it is unlikely that this error is very significant.

Numerical error is caused by the hardware running the simulation and can be reduced with higher precision variables to store information, although this was beyond the scope of this project, and as such has not been addressed.

The final possible limitation that could arise is the value used for the yield strength of magnesium alloy. Because Solidworks material library does not have a value for magnesium alloy's yield strength, it was inputted manually referencing existing data. However, magnesium alloys vary in composition, and as such the values range from 130 to 285MPa. The value used was 130MPa to test for failure with the worst possible value.

Total word count excluding tables, figures, references and bibliography: 2157

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